AN A PRIORI ESTIMATE FOR THE SINGLY PERIODIC
SOLUTIONS OF A SEMILINEAR EQUATION

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Abstract. We consider the positive solutions $u$ of $-\Delta u + u - u^p = 0$ in $[0, 2\pi] \times \mathbb{R}^{N-1}$, which are $2\pi$-periodic in $x_1$ and tend uniformly to 0 in the other variables. There exists a constant $C$ such that any solution $u$ verifies $u(x_1, x') \leq Cw_0(x')$ where $w_0$ is the ground state solution of $-\Delta v + v - v^p = 0$ in $\mathbb{R}^{N-1}$. We prove that exactly the same estimate is true when the period is $2\pi/\varepsilon$, even when $\varepsilon$ tends to 0. We have a similar result for the gradient.

Key words:
35B10 35B32 35B40 35G30 35J40

1. Introduction.

Let $N$ be an integer, $N \geq 2$, let $\varepsilon$ and $p$ be positive real numbers, $p > 1$. We study the equation

\begin{equation}
-\varepsilon^2 \Delta u + u - u^p = 0 \text{ in } S^1 \times \mathbb{R}^{N-1}
\end{equation}

where $S^1 = [0, 2\pi]$. We mean that $u$ is $2\pi$-periodic in $x_1$. We consider the positive solutions of (1.1), $u(x_1, x')$ ($x_1 \in S^1$ and $x' \in \mathbb{R}^{N-1}$). In the whole paper, the solutions of (1.1) are supposed to tend to 0 as $\|x'\|$ tends to infinity, uniformly in $x_1$. It is known that these solutions are radial in $x'$ and decreasing in $\|x'\|$. This can be proved by an application of the moving plane method ([11], [5], [2], [9]). The ground-state solution $w_0$ in $\mathbb{R}^{N-1}$, which is defined when $1 < p < (N + 1)/(N - 3)$, is radial on $\mathbb{R}^{N-1}$ and is a particular solution of (1.1) which does not depend on $x_1$. In [3], Dancer proved the existence of positive solutions really depending on $x_1$ and $x'$. In [1], we studied the case $N = 2$ and we proved some properties of the curves of solutions of (1.1) which are obtained by bifurcation from the ground state $w_0$. We have need some exponential estimates on the solutions of (1.1), which decay to 0 in $\|x'\|$, uniformly in $x_1$. There, it was sufficient for our purpose to prove estimates which are uniform only for $\varepsilon$ far from 0.

We thought it could be interesting to establish uniformly valid a priori estimates, even when $\varepsilon$ tends to 0.

In this paper we suppose that

\begin{equation}
1 < p < \frac{N + 2}{N - 2}
\end{equation}

If $N = 2$, this condition is $p > 1$. 

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We know, see [3], that the condition (1.2) for $p$ is a necessary and sufficient condition to have the following property: There exists $M > 0$ such for all $\varepsilon > 0$, any positive solution $u$ of (1.1) verifies

$$\|u\|_{L^\infty} \leq M$$

This property is related to the nonexistence of positive solutions for the equation $-\Delta u - u^p = 0$, more precisely

$$(v \geq 0, -\Delta v - v^p = 0 \text{ in } \mathbb{R}^N) \Rightarrow (v = 0)$$

(see Gidas and Spruck [6]).

This paper is devoted to the proof of the following theorem, concerning an exponential a priori estimate.

**Theorem 1.1.** There exists a real number $K$ independent of $\varepsilon > 0$ and of any solution $u$ of (1.1) which decays to 0 as $\|x\| \to \infty$ uniformly in $x_1$, such that for all $x = (x_1, x')$ in $S^1 \times \mathbb{R}^{N-1}$, we have, with $r' = \|x\|$, 

$$u(x) \leq Ke^{-\frac{\varepsilon}{2} \left( \frac{r'}{\varepsilon} \right)^{\frac{2-N}{2}}}$$

(1.5)

$$\|\nabla u(x)\| \leq K\varepsilon e^{-\frac{\varepsilon}{2} \left( \frac{r'}{\varepsilon} \right)^{\frac{2-N}{2}}}$$

(1.6)

In all what follows we will use $\tilde{u}(x_1, x') = u(\varepsilon x_1, \varepsilon x')$ for $(x_1, x') \in S^1/\varepsilon \times \mathbb{R}^{N-1}$. The inequality (1.5) means that there exists $C$ independent of $\varepsilon$ such that $\tilde{u}(x) \leq C \varepsilon_0(x')$.

The technique we use in the proof of Theorem 1.1 is inspired by Gidas-Ni-Nirenberg [5]. But instead of a null limit at infinity, we have a $2\pi/\varepsilon$ periodicity in $x_1$, and we obtain bounds which are independent of the period.

First, we prove that, as $\|x\| \to \infty$, the limit to 0 of the solutions under consideration is uniform in $\varepsilon$. For this purpose, we use some known result for $N = 2$ concerning the dependence on only one variable for some solutions which are monotone in one of the variables (Ghoussoub-Gui [8], Farina [4]).

2. **Proof of Theorem 1.1**

We begin the proof by two propositions. The first one immediately follows from [8] (see also [4] for larger hypothesis).
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Proposition 2.1. Let \( v \) be a bounded solution of
\[
-\Delta v + v - v^p = 0 \quad \text{in} \quad \mathbb{R}^2
\]
Let us suppose that \( \frac{\partial v}{\partial x_i} \) is bounded and \( \frac{\partial v}{\partial x_i} \leq 0 \) in \( \mathbb{R}^2 \), for \( i = 1 \) or \( i = 2 \). Then \( v \) does not depend on the variable \( x_i \).

Proof: From [8], Theorem 1.1, there exist a function \( U \) and a vector \( a \in \mathbb{R}^2 \) such that \( v(x) = U(a.x) \). We have
\[
-\|a\|^2 U''(a.x) + (U - U^p)(a.x) = 0
\]
If \( a_i \neq 0 \), \( U \) is monotone and bounded in \( \mathbb{R} \). The only possibility is that \( U \) is a constant function, equal to 0 or 1, so is \( v \).

Proposition 2.2. Let \( (\varepsilon, u) \) be solutions of (1.1). Then \( \tilde{u}(x_1, r) \) tends to 0 as \( r \) tends to \( \infty \), uniformly with respect to \( x_1 \) and to \( \varepsilon \) and \( u \).

Proof: In this proof, we omit the indices of the sequences. Let us suppose, by contradiction, that there exist a sequence \( (a, b) \in \mathbb{R}^N \), with \( ||b|| \) tending to \( +\infty \), a real positive number \( \varepsilon_2 \) and solutions \( (\varepsilon, u) \) of (1.1) such that \( \tilde{u}(a, b) \geq \varepsilon_2 \). We can suppose \( \varepsilon_2 < 1 \). For every solution \( (\varepsilon, u) \), we have that \( \lim_{r \to \infty} \tilde{u}(x_1, r) = 0 \), uniformly in \( x_1 \). So, for every \( \varepsilon_1 \in ]0, \varepsilon_2[ \), there exists a sequence, \( \tilde{b} \), \( ||\tilde{b}|| \geq ||b|| \), such that \( \tilde{u}(a, \tilde{b}) = \varepsilon_1 \). With the same argument, we define a sequence, still denoted by \( b \), with \( ||b|| \) tending to \( \infty \), such that \( \tilde{u}(a, b) = \varepsilon_2 \). As \( \tilde{u} \) is radial in \( x' \), let us define
\[
v(x_1, r) = \tilde{u}(x_1 + a, r + ||b||) \quad \text{for} \quad r \geq -||b||
\]
and
\[
\tau(x_1, r) = \tilde{u}(x_1 + a, r + ||\tilde{b}||) \quad \text{for} \quad r \geq -||\tilde{b}||
\]
The function \( v \) verifies
\[
-v_{x_1 x_1} - v_{rr} - \frac{N - 2}{r + ||b||} v_r + v - v^p = 0
\]
and \( \tau \) verifies a similar equation. It is standard that both sequences \( v \) and \( \tau \) tend uniformly on the compact sets of \( \mathbb{R}^2 \) to limits, which will be denoted respectively by \( z \) and \( \tau \). But \( z \) and \( \tau \) are positive, bounded and non increasing in the variable \( r \) and they are periodic in \( x_1 \). Moreover, \( z \) and \( \tau \) verify
\[
-z_{x_1 x_1} - z_{rr} + z - z^p = 0 \quad \text{in} \quad \mathbb{R}^2
\]
By Proposition 2.1, \( z \) and \( \tau \) depend only on \( x_1 \). By Kwong, [10], if they are not constant functions, they oscillate indefinitely as \( x_1 \) tends to \( \infty \), around the solution 1. As \( 0 < \varepsilon_1 < \varepsilon_2 < 1 \), then \( \tau \) and \( z \) are not constant solutions. So \( z \) and \( \tau \) oscillate infinitely around 1, too. The function \( h = z - 1 \) and the function \( \tilde{h} = \tau - 1 \) verify respectively the equations
\[
(2.8) \quad h'' + h(-1 + \frac{z^p - 1}{z - 1}) = 0 \quad \text{and} \quad \tilde{h}'' + \tilde{h}(-1 + \frac{\tau^p - 1}{\tau - 1}) = 0
\]
As \( z \geq \tilde{\tau} \) and \( z(0) > \tau(0) \), we have from the ordinary differential equations theory that \( z > \tau \). It is easy to see that \(-1 + \frac{z^p - 1}{z - 1} > -1 + \frac{\tau^p - 1}{\tau - 1} \). By the Sturm Theory
(see Ince, quoted in [10], Lemma 1), applied to the equations (2.8), there exists at least one zero of \( z - 1 \) between any two consecutive zeroes of \( z - 1 \). But there exist pairs \((\alpha, \beta)\) of zeroes of \( z - 1 \) such that \( z > 1 \) in \([\alpha, \beta]\). Thus \( z > 1 \) in \([\alpha, \beta]\). We get a contradiction. We infer that the sequence \((a, b)\), in the beginning of this proof, doesn’t exist. We have proved the proposition.

We will need an uniform bound for the gradient of the solutions.

**Remark 2.1.** There exists \( M \), such that for all solution \((\varepsilon, u)\) of (1.1)

\[
\|\nabla \tilde{u}\|_{L^\infty([s_1^1/\varepsilon, R^{N-1})} \leq M
\]

**Proof** Let \((a, b) \in (S^1/\varepsilon) \times \mathbb{R}^{N-1}\). We set \( v(x_1, x') = \tilde{u}(x_1 + a, x' + b) \). It verifies \(-\Delta v + v - v^p = 0\) in \((S^1/\varepsilon) \times \mathbb{R}^{N-1}\). Moreover, we have \( \|v\|_\infty \leq M \), for a constant \( M \) independent from \( \varepsilon \). By standard elliptic arguments, [7], \( \nabla v \) is bounded on the compact sets of \( \mathbb{R}^N \). So, there exists \( M \), independent from \( \varepsilon \), such that for all \( x_1 \in S^1/\varepsilon \)

\[
(2.10) \quad \tilde{u}^{p-1}(x_1, r') < \eta
\]

Integrating (1.1) with respect to \( x_1 \), we find for \( r' > X \)

\[
(2.11) \quad h_{r'} + ((N - 2)/r')h_r > (1 - \eta)h
\]

Let us multiply (2.11) by \( h_r \), we obtain that the function \( h_r^2 - (1 - \eta)h^2 \) is non increasing. Moreover, it tends to 0 as \( r' \) tends to \( \infty \). We get \( h_r + \sqrt{1 - \eta}h \leq 0 \), for \( r' > X \). So there exists \( C \) such that for all \( r' \)

\[
(2.12) \quad h(r') \leq \frac{C}{\varepsilon} e^{-\sqrt{1 - \eta}r'}
\]

Let us remark that the constant \( C \) is independent from the choice of the solution \((\varepsilon, u)\).

Let \( R > 0 \) be a given positive real number. We use a Harnack inequality ([7], Theorem 9.20) to get that there exists a constant \( C \) independent from \( y \) and from \( \varepsilon \) such that

\[
(2.13) \quad \sup_{B_R(y)} \tilde{u} \leq C \int_{B_{2R}(y)} \tilde{u} \leq C \int_{\|x'-y'\| \leq 2R} \int_{y_1 - R}^{y_1 + R} \tilde{u}(x_1, x')dx_1dx'
\]
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that gives
\[ \sup_{B_R(y)} \tilde{u} \leq C \int_{\|x'-y\| \leq 2R} h(\|x'\|)dx'. \]

Finally, using (2.12), for all \( \eta \in [0, 1[ \) there exists \( C \), independent from the solution \( (\varepsilon, u) \), such that,
\begin{equation}
\tilde{u}(y) \leq \frac{C}{\varepsilon} e^{-\eta\|y\|}.
\end{equation}

For the remainder of the proof, we will need the Green function for the equation (1.1). The notation \( \Delta' \) will stand for the Laplacian operator in \( \mathbb{R}^{N-1} \). We have
\begin{equation}
G(x_1, x') = \sum_{j=0}^{\infty} k_j^{N-3} \frac{k_j}{\varepsilon^{N-1}} g\left(\frac{k_j}{\varepsilon} x' \right) \cos(jx_1)
\end{equation}
where \( k_j = \sqrt{1 + \varepsilon^2 j^2} \) and \( g \) is the Green function for the operator \(-\Delta' + I\) in \( \mathbb{R}^n \), \( n = N - 1 \), with the null limit at infinity. It is recalled in [5] that
\begin{equation}
0 < g(r) \leq \frac{C e^{-r}}{r^{n-2}} (1 + r)^{(n-3)/2} \text{ for } n \geq 2 \text{ and } g(r) = \frac{1}{2} e^{-r} \text{ for } n = 1
\end{equation}
We will need the following estimate, valid for all \( \eta \in [0, 1[ \).
\begin{equation}
\int_{\mathbb{R}^{N-1}} g(||y' - x'||) e^{-\eta\|y'||} dy' \leq C e^{-\eta\|x'||}
\end{equation}
which is an easy consequence of (2.16). For all function \( f \), that is \( 2\pi \)-periodic in \( x_1 \), the solution of
\[-\varepsilon^2 \Delta u + u = f \text{ in } \mathbb{R}^N\]
that is \( 2\pi \)-periodic in \( x_1 \) and that tends to 0, as \( \|x'\| \) tends to \( \infty \) is \( u = G \ast f \). If \( f \) is positive, then \( u \) is positive, by the maximum principle. So \( G \) is positive. Moreover we can use (2.15) to verify that
\begin{equation}
\int_{S^1} G(x_1, x') dx_1 = \frac{2\pi}{\varepsilon^{N-1}} g\left(\frac{x'}{\varepsilon}\right)
\end{equation}
Let us prove that for all \( \eta \in [0, 1[ \), there exists \( C \), independent from \( x_1 \) and from \( (\varepsilon, u) \) such that
\begin{equation}
\tilde{u}(x_1, x') \leq C e^{-\eta r'}
\end{equation}
It is clear by (2.14) that for all solution \( (\varepsilon, u) \) and all \( \eta \in [0, 1[ \), the function \( \tilde{u} e^{\eta r} \) belongs to \( L^\infty(\mathbb{R}^N) \). We set
\[ K(\eta) = \|\tilde{u} e^{\eta r'}\|_\infty \]
We use the Green function \( G \) to get
\begin{equation}
u(x_1, x') = \int_{S^1 \times \mathbb{R}^{N-1}} G(y_1 - x_1, y' - x') u_p(y_1, y') dy_1 dy'
\end{equation}
and (2.18) gives
\[ \tilde{u}(x_1, x') \leq 2\pi K\left(\frac{\eta}{p}\right)^p \int_{\mathbb{R}^{N-1}} g(||y' - x'||) e^{-\eta||y'||} dy' \]

By (2.17), we infer that there exists a constant \( C \), independent from \((\varepsilon, u)\), such that
\[ (2.21) \quad K(\eta) \leq CK\left(\frac{\eta}{p}\right)^p \]

Now, let \(\tau = (\tau_1, \tau')\) be such that the function \(\tilde{u}(x + \tau)e^{\eta||x' + \tau'||}\) attains its maximal value at \(x = 0\). The existence of \(\tau\) is provided by (2.14). Let us suppose that \(K(\eta)\) tends to \(\infty\). We claim that \(||\tau'||\) tends to infinity. Let us prove this claim. Let \(\alpha\) be a positive real number, that will be chosen later. We set
\[ v(x) = \tilde{u}(\alpha x + \tau)e^{\eta||\alpha x' + \tau'||}/K(\eta) \]

It verifies
\[ -\Delta v + \left(1 + \eta^2 + \frac{(N - 2)\eta}{||\alpha x' + \tau'||}\right) \alpha^2 v = K(\eta)^{p-1} \alpha^2 e^{(-p+1)\eta||\alpha x' + \tau'||}v + \frac{2\eta\alpha^2}{K(\eta)} \sum_{i=2}^N \frac{\partial \tilde{u}}{\partial x_i}(\alpha x + \tau)\left(\frac{\alpha x_i + \tau_i}{||\alpha x' + \tau'||}\right) e^{\eta||\alpha x' + \tau'||} \]

If \(||\tau'||\) were bounded, we would choose \(\alpha\) that tends to 0 such that \(K(\eta)^{p-1} \alpha^2 e^{(-p+1)\eta||\alpha x' + \tau'||}\) tends to 1. By Lemma 2.1 and by standard results, \(v\) would tend to a limit \(\overline{v}\), uniformly in the compact sets of \(\mathbb{R}^N\). Then, \(\overline{v}\) would verify \(-\Delta \overline{v} - \overline{v}^p = 0\) while \(0 \leq \overline{v} \leq 1\) and \(\overline{v}(0) = 1\). This is impossible by (1.4). So, if we suppose that \(K(\eta)\) tends to \(\infty\), then \(||\tau'||\) tends to \(\infty\). Let \(\tilde{\tau} = (\tilde{\tau}_1, \tilde{\tau}')\) be such that \(K(\tilde{\tau}) = \tilde{u}(\tilde{\tau})e^{2\eta||\tilde{\tau}'||}\).

We have \(K(\tilde{\tau}) = \tilde{u}(\tilde{\tau})e^{2\eta||\tilde{\tau}'||}\), that gives
\[ (2.22) \quad K\left(\frac{\eta}{p}\right)^p \leq K(\eta)\tilde{u}^{p-1}(\tilde{\tau}) \]

Then (2.21) and (2.22) give
\[ (2.23) \quad K(\eta) \leq CK\left(\frac{\eta}{p}\right)^p \leq C(\eta)\tilde{u}^{p-1}(\tilde{\tau}) \]

Consequently, if \(K(\eta)\) tends to \(\infty\), then \(K(\tilde{\tau})\) tends to \(\infty\), too. Then, \(||\tilde{\tau}'|| \to \infty\).

By Proposition 2.2, we have \(\tilde{u}(\tilde{\tau}) \to 0\). Then (2.23) gives a contradiction. So, we have proved that for all \(\eta \in ]0, 1[\), \(K(\eta)\) is bounded, independently from \((\varepsilon, u)\). We have (2.19).

Now, let us choose \(\eta\) such that \(\eta p > 1\). In [5], it is proved that for \(b > 1\) and for \(N - 1 \geq 2\)
\[ \int_{\mathbb{R}^{N-1}} g(||x' - y'||) e^{-b||y'||} dy' \leq C||x'||^{2-N} e^{-||x'||} \]
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We can use (2.16) to prove that the estimate (2.24) is valid also for \( N = 2 \). Now we use (2.20), (2.18) and (2.24) to obtain (1.5) with \( K \) independent from \((\varepsilon, u)\). Now, let us estimate the gradient of \( u \). We have, for \( i = 1, \ldots, N \)

\[
(2.25) \quad \frac{\partial u}{\partial x_i}(x_1, x') = p \int_{S^1 \times \mathbb{R}^{N-1}} G(y_1 - x_1, y' - x')(u^{p-1} \frac{\partial u}{\partial x_i})(y_1, y')dy_1dy'
\]

Since \( \frac{\partial u}{\partial x_i} \) is bounded and \( u \leq Ce^{-y'/\varepsilon} \), that gives

\[
| \frac{\partial u}{\partial x_i}(x_1, x') | \leq \frac{C}{\varepsilon} \int_{S^1 \times \mathbb{R}^{N-1}} G(y_1 - x_1, y' - x')e^{-(p-1)\|y'\|/\varepsilon}dy_1dy'
\]

and (2.18) gives

\[
(2.26) \quad | \frac{\partial \tilde{u}}{\partial x_i}(x_1, x') | \leq C \int_{\mathbb{R}^{N-1}} g(\|y' - x'\|)e^{-(p-1)\|y'\|}dy'
\]

Now, the proof is more easy if \( p > 2 \) than if \( p < 2 \). If \( p > 2 \), we deduce directly (1.6) from (2.24) and (2.26). If \( 1 < p < 2 \), we deduce from (2.17) and (2.26) that

\[
| \frac{\partial \tilde{u}}{\partial x_i}(x_1, x') | \leq Ce^{-(p-1)\|x'\|}
\]

Iterating this process, we get an integer \( k \) such that

\[
| \frac{\partial \tilde{u}}{\partial x_i}(x_1, x') | \leq Ce^{-k(p-1)\|x'\|}
\]

with \( k(p-1) < 1 \) and \((k+1)(p-1) \geq 1 \). If \((k+1)(p-1) > 1 \), we get (1.6) and the proof is complete. If \((k+1)(p-1) = 1 \), we get

\[
(2.27) \quad | \frac{\partial \tilde{u}}{\partial x_i}(x_1, x') | \leq C \int_{\mathbb{R}^{N-1}} g(\|y' - x'\|)e^{-\|y'\|}dy'
\]

If \( N \geq 3 \), we have

\[
\int_{\mathbb{R}^{N-1}} g(\|y' - x'\|)e^{-\|y'\|}dy' \leq C \int_{\mathbb{R}^{N-1}} e^{-\|x' - y'\| - \|y'\|}(1 + \|x' - y'\|)^{(N-4)/2}/\|x' - y'\|^N}dy'
\]

We can write the integral in the right hand member of this inequality as \( I = I_1 + I_2 \) and

\[
I_1 = \int_{\|z\| \leq \|x'\|} e^{-\|z\| - \|x' + z\|}((1 + \|z\|)^{(N-4)/2})/\|z\|^N}dz
\]

and

\[
I_2 = \int_{\|z\| \geq \|x'\|} e^{-\|z\| - \|x' + z\|}((1 + \|z\|)^{(N-4)/2})/\|z\|^N}dz
\]

We obtain, as \( \|x'\| \) tends to \( \infty \),

\[
I_1 \leq e^{-\|x'\|} \int_0^{\|x'\|} (1 + s)^{N-4}/s^2 ds \quad \text{and} \quad I_2 \leq e^{\|x'\|} \int_{\|x'\|}^{+\infty} e^{-2s}(1 + s)^{N-4}/s^2 ds
\]
These integrals are both less than \( Ce^{-\|x'\|} \|x'\|^\frac{N}{2} \). Thus, if \( N \geq 3 \) we have obtained that
\[
(2.28) \quad \left| \frac{\partial \tilde{u}}{\partial x_i} (x_1, x') \right| \leq C e^{-\|x'\|} \|x'\|^\frac{N}{2}
\]
If \( N = 2 \), we have, when \( |x'| \) tends to \( \infty \)
\[
\int_{\mathbb{R}} g(|y' - x'|) e^{-|y'|} dy' \leq C \int_{\mathbb{R}} e^{-|x' - y'| - |y'|} dy' \leq C |x'| e^{-|x'|}
\]
In any case, we get that there exists \( b \in ]0, 1[ \), with \( b + p - 1 > 1 \) and such that
\[
| \frac{\partial \tilde{u}}{\partial x_1} (x_1, x') | \leq C e^{-b|x'|}
\]
Using this estimate in (2.25) and thanks to (2.24), we get (1.6), for \( 1 < p < 2 \). If \( p = 2 \), (2.26) is (2.27) and we deduce (2.28) again. This ended the proof of Proposition 1.1.

References


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