Gelfand Numbers and Euclidean Sections of Large Dimensions

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Let $E = (\mathbf{R}^n, \|\cdot\|)$ be an n-dimensional Banach space and let B_E be the unit ball in E. We shall also consider a Euclidean structure on $\mathbf{R}^n$, and so, let $(\cdot,\cdot)$ denote an inner product and $\|\cdot\|_2$ the corresponding Euclidean norm. The dual space E^* is naturally identified to $(\mathbf{R}^n, \|\cdot\|_*)$, where

$$\|x\|_* = \sup\{|(x,y)| \mid y \in B_E\}. \tag{1}$$

Let B_2^n be the Euclidean unit ball and let $S = S_{n-1}$ be the unit sphere. Let μ be the normalized rotation invariant measure on S. Set

$$M(E) = \left(\int_S \|x\|^2 d\mu\right)^{1/2}, \quad M(E^*) = \left(\int_S \|x\|_*^2 d\mu\right)^{1/2}. \tag{2}$$

Let us recall the main result from [P-T], which was motivated by study of V. Milman (cf. [M.1], [M.2]).

Theorem 1. *Let $E = (\mathbf{R}^n, \|\cdot\|)$ and let $\|\cdot\|_2$ be a Euclidean norm on $\mathbf{R}^n$. For every integer k, $1 \le k \le n$, there exists a subspace F of $\mathbf{R}^n$ with $\dim F = n-k$ such that*

$$\|x\|_2 \le KM(E^*)(n/k)^{1/2}\|x\| \quad \textit{for} \quad x \in F,$$

where K is a universal constant.

The main technical result of this note improves Theorem 1. In particular, it will enable us to obtain precise estimates in some cases of a

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