A note on subgaussian estimates for linear functionals on convex bodies

A. Giannopoulos* A. Pajor† and G. Paouris‡

Abstract

We give an alternative proof of a recent result of Klartag on the existence of almost subgaussian linear functionals on convex bodies. If K is a convex body in $\mathbb{R}^n$ with volume one and center of mass at the origin, there exists $x \neq 0$ such that

$$|\{y \in K : |\langle y, x\rangle| \geqslant t\|\langle \cdot, x\rangle\|_1\}| \leqslant \exp(-ct^2/\log^2(t+1))$$

for all $t \geqslant 1$, where $c > 0$ is an absolute constant. The proof is based on the study of the L_q–centroid bodies of K. Analogous results hold true for general log-concave measures.

1 Introduction

The purpose of this note is to provide an alternative proof of a recent result of Klartag (see [9]) on the existence of almost subgaussian linear functionals on convex bodies. Let K be a convex body in $\mathbb{R}^n$ with volume $|K| = 1$ and center of mass at the origin. Let $\psi : [0, \infty) \to [0, \infty)$ be a convex, increasing function with $\psi(0) = 0$. For every bounded measurable function $f : K \to \mathbb{R}$, define

$$\|f\|_\psi = \inf \left\{ t > 0 : \int_K \psi(|f(x)|/t)\, dx \leqslant 1 \right\}. \tag{1.1}$$

We will be interested in the ψ_α–norm of linear functionals $y \mapsto \langle y, x\rangle$ on K, where $1 \leqslant \alpha \leqslant 2$ and $\psi_\alpha(t) = e^{t^\alpha} - 1$. We say that $x \neq 0$ defines a ψ_α–direction for K with constant $B > 0$ if

$$\|\langle \cdot, x\rangle\|_{\psi_\alpha} \leqslant B\|\langle \cdot, x\rangle\|_1. \tag{1.2}$$

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It is not hard to check that this holds true if and only if

$$\|\langle\cdot,x\rangle\|_q \leqslant cBq^{1/\alpha}\|\langle\cdot,x\rangle\|_1 \tag{1.3}$$

for every $q \geqslant 1$, where $c > 0$ is an absolute constant. By Borell's lemma (see [13], Appendix III), there exists an absolute constant $C > 0$ such that if K is a convex body in $\mathbb{R}^n$, then every $x \neq 0$ is a ψ_1–direction for K with constant C.

The study of ψ_2–directions for linear functionals on convex bodies is motivated by the study of isotropic convex bodies and Bourgain's approach to the isotropic constant problem. A convex body K in $\mathbb{R}^n$ is called isotropic if it has volume $|K| = 1$, center of mass at the origin, and there exists a constant $L_K > 0$ such that

$$\int_K \langle y,\theta\rangle^2 dy = L_K^2 \tag{1.4}$$

for every $\theta \in S^{n-1}$. Every convex body with center of mass at the origin has a linear image which is isotropic (see [12]). This image is unique up to orthogonal transformations, and hence, the isotropic constant L_K is well–defined for the linear class of K. The isotropic constant problem asks if there exists an absolute constant $C > 0$ such that $L_K \leqslant C$ for every isotropic convex body in any dimension. One can easily see that $L_K = O(\sqrt{n})$ for every K. Uniform boundedness of L_K is known for some classes of bodies: unit balls of spaces with 1–unconditional basis, zonoids and their polars, etc. Bourgain (see [4]) proved that $L_K = O(\sqrt[4]{n}\log n)$ and, very recently, Klartag (see [8]) improved this bound to $L_K = O(\sqrt[4]{n})$. Moreover, in [5] Bourgain proved that if every $x \neq 0$ is a ψ_2–direction for K with constant B, then L_K is bounded by $cB\log(B+1)$.

A question of Milman, related to this line of thought, is whether, for every isotropic convex body K in $\mathbb{R}^n$, most $\theta \in S^{n-1}$ define a ψ_2–direction for K with a "good" constant (for example, logarithmic in n). Until recently, it was not known if there exists an absolute constant $C > 0$ such that every isotropic convex body has at least one ψ_2–direction with constant C. Some positive results are known for special classes of convex bodies. Bobkov and Nazarov (see [2] and [3]) have proved that if K is an isotropic 1–unconditional convex body, then $\|\langle\cdot,x\rangle\|_{\psi_2} \leqslant c\sqrt{n}\|x\|_\infty$ for every $x \neq 0$. This shows that the diagonal direction is a ψ_2–direction. For the class of zonoids, the existence of good ψ_2–directions was established in [14]. Another partial result, which gives more information in the case of isotropic convex bodies with "small diameter", was obtained in [15]: If $K \subseteq (\gamma\sqrt{n}L_K)B_2^n$ for some $\gamma > 0$, then

$$\sigma\big(\theta \in S^{n-1} : \|\langle\cdot,\theta\rangle\|_{\psi_2} \geqslant c_1\gamma tL_K\big) \leqslant \exp(-c_2\sqrt{n}t^2/\gamma) \tag{1.5}$$

for every $t \geqslant 1$, where σ is the rotationally invariant probability measure on S^{n-1} and $c_1, c_2 > 0$ are absolute constants.

Klartag (see [9]) gave a positive answer to this question, showing that every isotropic convex body admits at least one almost subgaussian linear functional. Our aim is to give a second (short) proof of this fact.

Theorem 1.1. *Let K be an isotropic convex body in $\mathbb{R}^n$. There exists $x \neq 0$ such that*

$$|\{y \in K : |\langle y, x\rangle| \geqslant t\|\langle \cdot, x\rangle\|_1\}| \leqslant \exp(-ct^2/\log^\tau(t+1)) \tag{1.6}$$

for all $t \geqslant 1$, where $c, \tau > 0$ are absolute constants.

It is clear that if x defines a ψ_α–direction for K and if $T \in SL(n)$, then T^*x defines a ψ_α–direction (with the same constant) for $T(K)$. It follows that Theorem 1.1 provides almost subgaussian directions for every convex body: If K is a convex body in $\mathbb{R}^n$ with volume one and center of mass at the origin, there exists $x \neq 0$ such that (1.6) holds true for all $t \geqslant 1$.

The argument of Klartag is based on the study of the level sets of the logarithmic Laplace transform of log–concave functions. The argument we present here is based on the study of the L_q–centroid bodies of an isotropic convex body. This family of bodies was studied and used by the third named author in [15], and in particular in [16], where the following sharp dimension–dependent concentration of volume estimate was proved: There exists an absolute constant $c > 0$ such that if K is an isotropic convex body in $\mathbb{R}^n$, then

$$\left|\left\{x \in K : \|x\|_2 \geqslant c\sqrt{n}L_K t\right\}\right| \leqslant \exp\left(-\sqrt{n}t\right) \tag{1.7}$$

for every $t \geqslant 1$, where $\|\cdot\|_2$ is the Euclidean norm. The tools which are developed in [16] allow us to give a very simple proof of Theorem 1.1. We present an argument which gives $\tau = 2$, i.e. the upper bound in (1.6) is $\exp(-ct^2/\log^2(t+1))$.

Notation. We work in $\mathbb{R}^n$, which is equipped with a Euclidean structure $\langle\cdot,\cdot\rangle$. We denote by $\|\cdot\|_2$ the corresponding Euclidean norm, and write B_2^n for the Euclidean unit ball, and S^{n-1} for the unit sphere. Volume is denoted by $|\cdot|$. If K is a convex body in $\mathbb{R}^n$, we set $\overline{K} = K/|K|^{1/n}$; this is the dilation of K which has volume one. We write σ for the rotationally invariant probability measure on S^{n-1}. The Grassmann manifold $G_{n,k}$ of k–dimensional subspaces of $\mathbb{R}^n$ is equipped with the Haar probability measure $\mu_{n,k}$.

A convex body is a compact convex subset C of $\mathbb{R}^n$ with non–empty interior. We say that C has center of mass at the origin if $\int_C \langle x, \theta\rangle dx = 0$ for every $\theta \in S^{n-1}$. The support function $h_C : \mathbb{R}^n \to \mathbb{R}$ of C is defined by $h_C(x) = \max\{\langle x, y\rangle : y \in C\}$. The mean width of C is defined by

$$w(C) = \int_{S^{n-1}} h_C(\theta)\, \sigma(d\theta). \tag{1.8}$$

The letters c, c', c_1, c_2 etc. denote absolute positive constants which may change from line to line. We refer to the books [18], [13] and [17] for basic facts from the Brunn–Minkowski theory and the asymptotic theory of finite dimensional normed spaces.

2 Normalized L_q–centroid bodies

Let K be a convex body of volume 1 in $\mathbb{R}^n$. For every $q \geqslant 1$ we define the L_q–centroid body $Z_q(K)$ of K by its support function:

$$h_{Z_q(K)}(x) = \|\langle \cdot, x\rangle\|_q := \left(\int_K |\langle y, x\rangle|^q dy \right)^{1/q}. \tag{2.1}$$

Since $|K| = 1$, we readily see that $Z_1(K) \subseteq Z_p(K) \subseteq Z_q(K) \subseteq Z_\infty(K)$ for every $1 \leqslant p \leqslant q \leqslant \infty$, where $Z_\infty(K) = \mathrm{conv}\{K, -K\}$. On the other hand, one has the reverse inclusions

$$Z_q(K) \subseteq \frac{cq}{p} Z_p(K) \tag{2.2}$$

for every $1 \leqslant p < q < \infty$, as a consequence of the ψ_1–behavior of $y \mapsto \langle y, x\rangle$. Observe that $Z_q(K)$ is always symmetric, and $Z_q(TK) = T(Z_q(K))$ for every $T \in SL(n)$ and $q \in [1, \infty]$. Also, if K has its center of mass at the origin, then $Z_q(K) \supseteq cZ_\infty(K)$ for all $q \geqslant n$, where $c > 0$ is an absolute constant.

It should be mentioned that L_q–centroid bodies were introduced in [10] under a different normalization. Lutwak, Yang and Zhang (see [11] and [7] for a different proof) have established the L_q affine isoperimetric inequality

$$|Z_q(K)|^{1/n} \geqslant |Z_q(\overline{B_2^n})|^{1/n} \geqslant c\sqrt{q/n} \tag{2.3}$$

for every $1 \leqslant q \leqslant n$, where $c > 0$ is an absolute constant.

We will need upper estimates for the quermassintegrals of the L_q–centroid bodies of an isotropic convex body. These follow immediately from estimates on the projections of $Z_q(K)$, which are obtained in [16]. Fix $1 \leqslant k \leqslant n$ and a k–dimensional subspace F of $\mathbb{R}^n$, and denote by E the orthogonal subspace of F. For every $\phi \in S_F$, define $E(\phi) = \{y \in \mathrm{span}\{E, \phi\} : \langle y, \phi\rangle \geqslant 0\}$. By a theorem of K. Ball (see [1] and [12]), for every convex body K of volume one in $\mathbb{R}^n$, for every $q \geqslant 0$ and every $\phi \in F$, the function

$$\phi \mapsto \|\phi\|_2^{1+\frac{q}{q+1}} \left(\int_{K\cap E(\phi)} |\langle y, \phi\rangle|^q dy \right)^{-\frac{1}{q+1}} \tag{2.4}$$

is a gauge function on F (see also [6] for the not necessarily symmetric case). If we denote by $B_q(K, F)$ the convex body in F whose gauge function is defined by (2.4), then the volume of $B_q(K, F)$ is given by

$$|B_q(K, F)| = |B_2^k| \int_{S_F} \left(\int_{K\cap E(\phi)} |\langle x, \phi\rangle|^q dx \right)^{\frac{k}{q+1}} d\sigma_F(\phi). \tag{2.5}$$

The following identity was proved in [16].

Proposition 2.1. *Let K be a convex body of volume 1 in $\mathbb{R}^n$ and let $1 \leqslant k \leqslant n-1$. For every $F \in G_{n,k}$ and every $q \geqslant 1$ we have that*

$$P_F(Z_q(K)) = (k+q)^{1/q}\,|B_{k+q-1}(K,F)|^{1/k+1/q} Z_q(\overline{B}_{k+q-1}(K,F)). \tag{2.6}$$

Using this identity and exploiting (2.5) in order to estimate the volume of $B_q(K,F)$, one gets the following estimate (see [16]).

Proposition 2.2. *Let K be an isotropic convex body in $\mathbb{R}^n$. If $F \in G_{n,k}$ and $E = F^{\perp}$ then, for every $q \in \mathbb{N}$ we have that*

$$P_F(Z_q(K)) \subseteq \frac{c(k+q)}{k} L_K Z_q(\overline{B}_{k+q-1}(K,F)) \tag{2.7}$$

where $c > 0$ is an absolute constant.

Definition 2.3. Let K be an isotropic convex body in $\mathbb{R}^n$. For every integer $q \geqslant 1$ we define the *normalized L_q–centroid body K_q* of K by

$$K_q = \frac{1}{\sqrt{q}L_K} Z_q(K). \tag{2.8}$$

Since $|Z_q(\overline{B}_{k+q-1}(K,F))| \leqslant |\overline{B}_{k+q-1}(K,F)| = 1$, Proposition 2.2 shows that

$$|P_F(K_q)|^{1/k} \leqslant \frac{c(k+q)}{k\sqrt{q}} |Z_q(\overline{B}_{k+q-1}(K,F))|^{1/k} \leqslant \frac{c_1(k+q)}{k}\frac{\sqrt{k}}{\sqrt{q}} |B_2^k|^{1/k} \tag{2.9}$$

for every $F \in G_{n,k}$. If $1 \leqslant k \leqslant q$, this estimate takes the simpler form

$$|P_F(K_q)|^{1/k} \leqslant 2c_1 \frac{\sqrt{q}}{\sqrt{k}} |B_2^k|^{1/k}. \tag{2.10}$$

In particular, for every $F \in G_{n,q}$ we have

$$|P_F(K_q)|^{1/k} \leqslant 2c_1 |B_2^k|^{1/k}. \tag{2.11}$$

A standard argument (based on the log–concavity of the quermassintegrals of $P_F(K_q)$) implies that since (2.11) is true for every $F \in G_{n,q}$, it remains valid for every $F \in G_{n,k}$, where $q \leqslant k \leqslant n$. We summarize these observations in the next Theorem.

Theorem 2.4. *Let K be an isotropic convex body in $\mathbb{R}^n$. If $1 \leqslant k, q \leqslant n$ are integers, and if $F \in G_{n,k}$, then*

$$|P_F(K_q)|^{1/k} \leqslant c_1 \max\{\sqrt{q/k}, 1\} |B_2^k|^{1/k}, \tag{2.12}$$

where $c_1 > 0$ is an absolute constant. In particular,

$$|K_q|^{1/n} \leqslant c_1 |B_2^n|^{1/n}. \tag{2.13}$$

The last ingredient of the proof is a consequence of the main result in [16]: from (1.7) it follows that

$$\left(\int_K \|y\|_2^q\,dy\right)^{1/q} \leqslant c\sqrt{n}L_K \tag{2.14}$$

for all $1 \leqslant q \leqslant \sqrt{n}$. Since

$$w(Z_q(K)) \leq \left(\int_{S^{n-1}}\int_K |\langle y,\theta\rangle|^q dy\,\sigma(d\theta)\right)^{1/q} \leqslant \left(\frac{C\sqrt{q}}{\sqrt{n}}\int_K \|y\|_2^q\,dy\right)^{1/q} \tag{2.15}$$

for all $1 \leqslant q \leqslant n$, we have the following Lemma.

Lemma 2.5. *Let K be an isotropic convex body in $\mathbb{R}^n$. If $1 \leqslant q \leqslant \sqrt{n}$, then*

$$w(K_q) \leqslant C, \tag{2.16}$$

where $C > 0$ is an absolute constant.

Remark 2.6. Without using Lemma 2.5, which fully exploits the results of [16], we can prove Theorem 1.1 with $\tau = 2 + \epsilon$ for any $\epsilon > 0$.

3 Covering numbers of K_q

Let $N(K_q, sB_2^n)$ denote the minimal number of translates of sB_2^n whose union covers K_q. A standard way to estimate the covering number $N(K_q, sB_2^n)$ is through the inequality

$$|tB_2^n| \cdot N(K_q, 2tB_2^n) \leqslant |K_q + tB_2^n|, \tag{3.1}$$

which is valid for every $t > 0$. We will use our information on the projections of K_q in order to give an upper bound for $|K_q + tB_2^n|$.

Proposition 3.1. *Let K be an isotropic convex body in $\mathbb{R}^n$. For every $1 \leqslant q \leqslant n$ and every $t > 0$, we have that*

$$N(K_q, 2tB_2^n) \leqslant \exp\left(C\frac{\sqrt{qn}}{\sqrt{t}} + C\frac{n}{t}\right), \tag{3.2}$$

where $C > 0$ is an absolute constant.

Proof. From the classical Steiner's formula we know that

$$|K_q + tB_2^n| = \sum_{k=0}^{n}\binom{n}{k}W_{[n-k]}(K_q)t^{n-k} \tag{3.3}$$

for all $t > 0$, where $W_{[n-k]}(K_q)$ is the mixed volume $V_k(K_q) = V(K_q; k, B_2^n; n-k)$ (see [18]).

We will use Kubota's integral formula to express $W_{[n-k]}(K_q)$ as an average of the volumes of the k–dimensional projections of K_q: for every $1 \leqslant k \leqslant n-1$ we have

$$W_{[n-k]}(K_q) = \frac{|B_2^n|}{|B_2^k|} \int_{G_{n,k}} |P_F(K_q)| \, d\mu_{n,k}(F). \tag{3.4}$$

Using (3.3), (3.4) and the estimates from Theorem 2.4, we can write

$$|K_q + tB_2^n| \leqslant |B_2^n| \sum_{k=0}^{n} \binom{n}{k} \left(c_1 \max\{\sqrt{q/k}, 1\} \right)^k t^{n-k}. \tag{3.5}$$

Then, (3.1) shows that

$$N(K_q, 2tB_2^n) \leqslant \sum_{k=0}^{q} \left(\frac{c_2 n \sqrt{q}}{k^{3/2} t} \right)^k + \sum_{k=q+1}^{n} \left(\frac{c_2 n}{kt} \right)^k. \tag{3.6}$$

Observe that for $1 \leqslant k \leqslant q$ we have

$$\left(\frac{c_2 n \sqrt{q}}{k^{3/2} t} \right)^k \leqslant \left(\frac{c_2 n q}{k^2 t} \right)^k \leqslant \frac{\left(c_3 \sqrt{nq/t} \right)^{2k}}{(2k)!}, \tag{3.7}$$

while, for $q \leqslant k \leqslant n$ we have

$$\left(\frac{c_2 n}{kt} \right)^k \leqslant \frac{(c_4 n/t)^k}{k!}. \tag{3.8}$$

It follows that

$$N(K_q, 2tB_2^n) \leqslant \exp\left(c_3 \frac{\sqrt{qn}}{\sqrt{t}} \right) + \exp\left(c_4 \frac{n}{t} \right), \tag{3.9}$$

and the result follows. $\square$

Remark 3.2. The proof actually gives $N(K_q, 2tB_2^n) \leqslant \exp\left(C \frac{n^{2/3} q^{1/3}}{t^{2/3}} + C \frac{n}{t} \right)$ for every $t > 0$, but this would play no role in the proof of the main result.

4 Proof of the Theorem

Let K be an isotropic convex body in $\mathbb{R}^n$. Consider the convex body

$$T = \mathrm{conv} \left(\bigcup_{i=1}^{\lfloor \log_2 n \rfloor} \frac{1}{i} K_{2^i} \right). \tag{4.1}$$

We will use the following standard fact.

Lemma 4.1. *Let $A_1, \ldots, A_s$ be subsets of RB_2^n. For every $t > 0$ we have that*

$$N(\mathrm{conv}(A_1 \cup \cdots \cup A_s), 2tB_2^n) \leqslant \left(\frac{cR}{t}\right)^s \prod_{i=1}^{s} N(A_i, tB_2^n). \tag{4.2}$$

Sketch of the proof. For $i = 1, \ldots, s$, let N_i be a subset of $\mathbb{R}^n$ with cardinality $|N_i| = N(A_i, tB_2^n)$, so that $A_i \subseteq \bigcup_{x_i \in N_i} (x_i + tB_2^n)$. Let B_1^s denote the unit ball of ℓ_1^s and fix $Z \subseteq B_1^s$ of minimal cardinality, so that $B_1^s \subseteq \bigcup_{z \in Z} (z + (t/R)B_1^n)$. It is well–known that $|Z| \leqslant (cR/t)^s$, where $c > 0$ is an absolute constant. Consider the set $N = \{w = z_1 x_1 + \cdots + z_s x_s : \; x_i \in N_i, z = (z_1, \ldots, z_s) \in Z\}$. Then, $\mathrm{conv}(A_1 \cup \cdots \cup A_s) \subseteq \bigcup_{w \in N} (w + 2tB_2^n)$. □

Let $s = \lfloor \log_2 n \rfloor$ and $m = \lfloor \log_2(\sqrt{n}) \rfloor \simeq s/2$. We apply Lemma 4.1 with $A_i = \frac{1}{i} K_{2^i}$, $1 \leqslant i \leqslant s$, and $t = 1$. Observe that $A_i \subseteq c_1 \sqrt{n} B_2^n$ for all $i \leqslant s$ (to see this, recall the known fact that if K is an isotropic convex body in $\mathbb{R}^n$, then $K \subseteq (cnL_K)B_2^n$). Using Sudakov's inequality (see [17]) and Lemma 2.5 to estimate $N(A_i, B_2^n)$ for $i \leqslant m$, and using the entropy estimates of Section 3 to estimate $N(A_i, B_2^n)$ for $m < i \leqslant s = \lfloor \log_2 n \rfloor$, we may write

$$\begin{aligned} N(T, B_2^n) &\leqslant (c_2 \sqrt{n})^{\lfloor \log_2 n \rfloor} \left[\prod_{i=1}^{\lfloor \log_2 n \rfloor} N(K_{2^i}, iB_2^n) \right] \\ &\leqslant e^{c_3 n} \exp\left(C\sqrt{n} \sum_{i=s+1}^{\lfloor \log_2 n \rfloor} 2^{i/2} \right) \times \exp\left(Cn \cdot \left(\sum_{i=1}^{m} \frac{1}{i^2} + \sum_{i=m+1}^{2m} \frac{1}{i} \right) \right) \\ &\leqslant e^{cn}. \end{aligned}$$

It follows that $|T| \leqslant |CB_2^n|$, where $C > 0$ is an absolute constant. Therefore, there exists $x \neq 0$ such that

$$h_T(x) \leqslant C\|x\|_2, \tag{4.3}$$

and hence,

$$\|\langle \cdot, x \rangle\|_{2^i} \leqslant C\, 2^{i/2} i L_K \|x\|_2 \tag{4.4}$$

for every $i = 1, 2, \ldots, \lfloor \log_2 n \rfloor$. This easily implies the following.

Theorem 4.2. *Let K be an isotropic convex body in $\mathbb{R}^n$. There exists $\theta \in S^{n-1}$ such that*

$$\|\langle \cdot, \theta \rangle\|_q \leqslant C\sqrt{q}\, \log q \|\langle \cdot, \theta \rangle\|_2 \tag{4.5}$$

for every $q \geqslant 2$, where $C > 0$ is an absolute constant.

A standard argument shows that Theorem 4.2 implies Theorem 1.1 (it is actually equivalent to Theorem 1.1 with $\tau = 2$).

Remark 4.3. The proof of Theorem 4.2 carries over to the case of an arbitrary log-concave measure: the approach of [16] and all the arguments we have used in this note depend only on the Brunn–Minkowski theory. It follows that if μ is an isotropic log–concave measure in $\mathbb{R}^n$, then there exists $\theta \in S^{n-1}$ such that

$$\|\langle\cdot,\theta\rangle\|_{L^q(\mu)} \leqslant C\sqrt{q}\,\log q\|\langle\cdot,\theta\rangle\|_{L^2(\mu)} \tag{4.6}$$

for all $2 \leqslant q \leqslant n$, where $C > 0$ is an absolute constant.

References

[1] K. M. Ball, *Logarithmically concave functions and sections of convex sets in* $\mathbb{R}^n$, Studia Math. **88** (1988), 69–84.

[2] S. G. Bobkov and F. L. Nazarov, *On convex bodies and log-concave probability measures with unconditional basis*, Geom. Aspects of Funct. Analysis (Milman-Schechtman eds.), Lecture Notes in Math. **1807** (2003), 53–69.

[3] S. G. Bobkov and F. L. Nazarov, *Large deviations of typical linear functionals on a convex body with unconditional basis*, Stochastic Inequalities and Applications, Progr. Probab. 56, Birkhauser, Basel, 2003, 3–13.

[4] J. Bourgain, *On the distribution of polynomials on high dimensional convex sets*, Geom. Aspects of Funct. Analysis (Lindenstrauss-Milman eds.), Lecture Notes in Math. **1469** (1991), 127–137.

[5] J. Bourgain, *On the isotropy constant for* ψ_2*-bodies*, Geom. Aspects of Funct. Analysis (Milman-Schechtman eds.), Lecture Notes in Math. **1807** (2003), 114–121.

[6] J. Bourgain, B. Klartag and V. D. Milman, *Symmetrization and isotropic constants of convex bodies*, Geom. Aspects of Funct. Analysis (Milman-Schechtman eds.), Lecture Notes in Math. **1850** (2004), 101–115.

[7] S. Campi and P. Gronchi, *The* L^p*-Busemann-Petty centroid inequality*, Adv. in Math. **167** (2002), 128–141.

[8] B. Klartag, *On convex perturbations with a bounded isotropic constant*, Geom. Funct. Anal. (2006), to appear.

[9] B. Klartag, *Uniform almost sub-gaussian estimates for linear functionals on convex sets*, Preprint.

[10] E. Lutwak and G. Zhang, *Blaschke-Santaló inequalities*, J. Differential Geom. **47** (1997), 1–16.

[11] E. Lutwak, D. Yang and G. Zhang, L^p *affine isoperimetric inequalities*, J. Differential Geom. **56** (2000), 111–132.

[12] V.D. Milman and A. Pajor, *Isotropic position and inertia ellipsoids and zonoids of the unit ball of a normed* n*-dimensional space*, Geom. Aspects of Funct. Analysis (Lindenstrauss-Milman eds.), Lecture Notes in Math. **1376** (1989), 64–104.

[13] V.D. Milman and G. Schechtman, *Asymptotic Theory of Finite Dimensional Normed Spaces*, Lecture Notes in Math. **1200** (1986), Springer, Berlin.

[14] G. Paouris, *Ψ_2-estimates for linear functionals on zonoids*, Geom. Aspects of Funct. Analysis (Milman-Schechtman eds.), Lecture Notes in Math. **1807** (2003), 211–222.

[15] G. Paouris, *On the Ψ_2-behavior of linear functionals on isotropic convex bodies*, Studia Math. **168** (2005), no. 3, 285–299.

[16] G. Paouris, *Concentration of mass on convex bodies*, Geom. Funct. Anal. (2006), to appear.

[17] G. Pisier, *The Volume of Convex Bodies and Banach Space Geometry*, Cambridge Tracts in Mathematics **94** (1989).

[18] R. Schneider, *Convex Bodies: The Brunn-Minkowski Theory*, Encyclopedia of Mathematics and its Applications **44**, Cambridge University Press, Cambridge (1993).

A. Giannopoulos: Department of Mathematics, University of Athens, Panepistimiopolis 157 84, Athens, Greece.

E-mail: apgiannop@math.uoa.gr

A. Pajor: Équipe d'Analyse et de Mathématiques Appliquées, Université de Marne-la-Vallée, Champs sur Marne, 77454, Marne-la-Vallée, Cedex 2, France.

E-mail: Alain.Pajor@univ-mlv.fr

G. Paouris: Équipe d'Analyse et de Mathématiques Appliquées, Université de Marne-la-Vallée, Champs sur Marne, 77454, Marne-la-Vallée, Cedex 2, France.

E-mail: grigoris_paouris@yahoo.co.uk