Sparse stochastic processes, fractals and wavelet analysis.

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Abstract

The "self-similar" processes described in this talk are generalized versions of the classical Lévy processes. They are generalized stochastic processes (in the sense Gelfand and Vilenkin) that are solutions of (unstable) fractional stochastic differential equations (fSDE). They are described by a general innovation model that is specified by: 1) a whitening operator (fractional derivative or Laplacian), which shapes their second-order moments, and 2) a Lévy exponent $f$, which controls the sparsity of the (non-Gaussian) innovations (white Lévy noise). We give a complete characterization these processes in terms of their characteristic form (the infinite-dimensional counterpart of the characteristic function). This allows us to prove that they admit a sparse representation in a wavelet bases. We also provide evidence that wavelets allow for a better $N$ term approximation than the classical Karhunen-Loève transform (KLT), except in the Gaussian case where the processes are equivalent to Mandelbrot’s fractional Brownian motion. We also highlight a fundamental connection with spline mathematics and the construction of maximally localized basis functions (B-splines).