Abstract

A result of Gallagher implies that for almost every \((\alpha, \beta) \in \mathbb{R}^2\)
\[
\liminf_{q \to \infty} q \log^2 q \|q\alpha\| \|q\beta\| = 0.
\]
In the first part of the talk I will try to convince you that this result can be improved and thus we can expect more from Littlewood’s Conjecture from a metrical point of view. In the second part, I will investigate concrete situations in which inhomogeneous Diophantine approximation results can be derived from their homogeneous counterparts. For example, for any \(i, j \geq 0\) with \(i + j = 1\) and \(\gamma \in \mathbb{R}\), let \(\text{Bad}_\gamma(i, j)\) denote the set of points \((x, y) \in \mathbb{R}^2\) for which
\[
\liminf_{q \to \infty} q \max\{\|qx - \gamma\|^{1/i}, \|qy - \gamma\|^{1/j}\} > 0.
\]
Then the basic construction that proves the homogeneous result that \(\dim \text{Bad}_\gamma(i, j) = 2\) can be naturally adapted to show that \(\dim \text{Bad}_\gamma(i, j) = 2\).