Scaling asymptotic properties of distributions and wavelet and non-wavelet transforms

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Abstract

We discuss several Tauberian aspects of wavelet and non-wavelet transforms of distributions. By a non-wavelet transform is meant \( F_\phi f(x, y) := (f * \phi_y)(x) \), where the kernel \( \phi \in \mathcal{S}(\mathbb{R}^n) \) satisfies \( \int_{\mathbb{R}^n} \phi(t)dt = 1 \) and \( \phi_y(t) = y^{-n} \phi(t/y^n) \). We present results that show that the scaling (weak) asymptotic properties of distributions are completely determined by angular boundary asymptotics of these transforms plus natural Tauberian hypotheses.

We present various applications. In particular, we define new classes of pointwise spaces and give their characterization through wavelet transforms. Such pointwise spaces generalize those introduced by Y. Meyer when defining pointwise scaling weak exponents and oscillating singularities of functions and distributions (Wavelets, vibrations and scalings, CRM Monograph series 9, AMS, Providence, 1998). We also discuss new results on the pointwise weak behavior of the family of Riemann-type distributions \( R_\beta(x) = \sum_{n=1}^{\infty} e^{i\pi x n^2}/n^{2\beta} \), \( \beta \in \mathbb{C} \), at rational points.

Furthermore, we present characterizations of positive measures in terms of angular inferior limits of non-wavelet transforms, and discuss how these ideas have recently led to the construction of a new integral, the distributional integral of functions of one real variable, which is more general than the Denjoy-Perron-Henstock integral, and in particular than that of Lebesgue.